



Nonlinear model predictive control of SOFC based on a Hammerstein model

Hai-Bo Huo^{a,*}, Xin-Jian Zhu^a, Wan-Qi Hu^b, Heng-Yong Tu^a, Jian Li^c, Jie Yang^d

^a Fuel Cell Research Institute, Shanghai Jiao Tong University, Shanghai 200240, China

^b Institute of Process Engineering, Chinese Academy of Sciences, Beijing 100080, China

^c School of Materials Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, Hubei Province, China

^d School of Mechanical and Electronic Engineering, China University of Geosciences, Wuhan 430074, Hubei Province, China

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ABSTRACT

To protect solid oxide fuel cell (SOFC) stack and meet the voltage demand of DC type loads, two control loops are designed for controlling fuel utilization and output voltage, respectively. A Hammerstein model of the SOFC is first presented for developing effective control strategies, in which the nonlinear static part is approximated by a radial basis function neural network (RBFNN) and the linear dynamic part is modeled by an autoregressive with exogenous input (ARX) model. As we know, the output voltage of the SOFC changes with load variations. After a primary control loop is designed to keep the fuel utilization as a steady-state constant, a nonlinear model predictive control (MPC) based on the Hammerstein model is developed to control the output voltage of the SOFC. The performance of the MPC controller is compared with that of the PI controller developed in [Y.H. Li, S.S. Choi, S. Rajakaruna, An analysis of the control and operation of a solid oxide fuel-cell power plant in an isolated system, IEEE Trans. Energy Convers. 20 (2) (2005) 381–387]. Simulation results demonstrate the potential of the proposed Hammerstein model for application to the control of the SOFC, while the excellence of the nonlinear MPC controller for voltage control of the SOFC is proved.

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1. Introduction

Distributed generation (DG) is a promising technology that can be used to address some of the technical as well as environmental concerns in power systems. As a kind of high-temperature fuel cell, solid oxide fuel cell (SOFC) presents an attractive option for the DG technology because it is modular, efficient and environmentally friendly. Unlike other types of fuel cells, the SOFC is entirely solid state with no liquid components. It usually works at a high temperature, in the range of 800–1000 °C to reach the electrolytes ionic conductivity requirement [1].

SOFC is a dynamic device which will affect the dynamic behavior of the power system to which it is connected. Analysis of such a behavior requires an accurate dynamic model. In the last several decades, fruitful results on modeling the nonlinear dynamics of the SOFC have been obtained [2–5]. However, most of these models emphasized the detailed description of cell internal processes, such as component material balance, energy balance and electrochemical kinetics, etc. These models are very useful to analyze the transient characteristics of the SOFC, but they are too com-

plicated to be used in controller design. For developing effective control strategies, Jurado et al. [6,7] have presented identification models for a SOFC. However, one of the most important cell performance variables, fuel utilization, has not been examined when the authors explored the SOFC dynamic response after the disturbances. Furthermore, they have not considered the fuel processor in their investigation.

Nonlinear input–output block-oriented models, such as Hammerstein and Wiener, do not require much fundamental knowledge about a system, and they are relatively easy to be constructed using process data. The Hammerstein model consists of a static nonlinear block followed in series by a dynamic linear block. It is a type of commonly used nonlinear models, and has been successfully used to model a class of nonlinear systems [8–12]. The identification of the Hammerstein system involves estimating both the nonlinear and the linear blocks from the input–output observations. There exist a large number of identification methods for the Hammerstein model, and most assume the nonlinearity is a polynomial of finite and known order. However, if the nonlinearity is not a polynomial and the input is not Gaussian, these algorithms fail to converge [13]. To overcome the aforementioned deficiencies, Narendra and Parthasarathy [14] have pointed out that a neural network could be used as a nonlinear operator in a Hammerstein model. Compared with other types of artificial neural network (ANN), radial

* Corresponding author. Tel.: +86 21 34206249; fax: +86 21 34206249.
E-mail address: haibo761028@yahoo.com.cn (H.-B. Huo).

Nomenclature

E_0	ideal standard potential (V)
E	open-circuit reversible potential (V)
I_{FC}	stack current (A)
K_r	constant with the value of $N_0/4F$ ($\text{mol s}^{-1} \text{A}^{-1}$)
K_i	valve molar constants for hydrogen, oxygen and water ($\text{mol s}^{-1} \text{atm}^{-1}$)
$N_{H_2}^r$	hydrogen reacted flow rate (mol s^{-1})
N_f	natural gas input flow (mol s^{-1})
N_0	number of cells in the stack
$N_{H_2}^{\text{in}}$	hydrogen input flow rate (mol s^{-1})
$N_{O_2}^{\text{in}}$	oxygen input flow rate (mol s^{-1})
$N_{H_2}^{\text{out}}$	hydrogen output flow rate (mol s^{-1})
p_i	partial pressures of hydrogen, oxygen, and water (atm)
r_{H_2O}	ratio of hydrogen to oxygen
r	ohmic loss (Ω)
T	stack operating temperature (K)
u	fuel utilization
u_s	desired utilization in steady state
V_{dc}	stack output voltage (V)
Greek letters	
τ_f	fuel processor response time (s)
τ_i	response times for the flow of hydrogen, oxygen and water (s)

basis function neural network (RBFNN) has a number of advantages, such as better approximation properties, simpler network structures and faster learning algorithms. To develop effective control strategies, a Hammerstein model of the SOFC is presented firstly, in which the nonlinear static part is approximated by a RBFNN and the linear dynamic part is modeled by an autoregressive with exogenous input (ARX) model. In addition, in the Hammerstein modeling the fuel processor is included and the operating issue about the fuel utilization is considered specifically.

Model predictive control (MPC) refers to a class of algorithms that compute a sequence of manipulated variable adjustments in

order to optimize the future behavior of a plant. Garcia et al. [15] have indicated that MPC was emerging as one of the most popular and effective control techniques in process industries. It has become an industry standard mainly due to its intrinsic capability for dealing with constraints and multivariable system.

SOFC produces DC electric power from fuel and oxidant via an electrochemical process. Any change in the load circuit or its demand for power changes the operating conditions for the SOFC. As a result, the output voltage of the SOFC will have a high fluctuation in response to the changes. Furthermore, if these changes cause the SOFC to work in the overused or underused fuel conditions, the actual performance of the FC will be affected. In order to meet the voltage demand of DC type loads and keep the fuel utilization constant by controlling the natural gas input flow, MPC scheme based on the Hammerstein model and a primary control loop are developed in this paper.

The rest of this paper is organized as follows. In Section 2, the SOFC dynamic model proposed in [5,16,17] is briefly reviewed. The detailed identification structure and identification algorithms of the Hammerstein model are given in Section 3. In Section 4, the MPC is formulated. Some simulation results are depicted to show the validity of the Hammerstein model and the practical applicability of the proposed control strategies to control the fuel utilization and the output voltage in Section 5. Finally, conclusions and suggestion for future work are presented in Section 6.

2. Theory for the dynamic model of SOFC

Based on the work reported in [5,16,17], the SOFC dynamic model is briefly reviewed in this section. The SOFC dynamic model including the fuel processor adopted in this paper is shown in Fig. 1 [16].

2.1. The balance of plant (BOP)

The BOP consists of the natural gas fuel storage, the fuel valve controlled by its controller, and the fuel processor that reforms the natural gas input N_f to the hydrogen-rich fuel $N_{H_2}^{\text{in}}$. In [5], the authors introduced a simple model of a fuel processor that converts fuels such as natural gas to hydrogen and byproduct gases. The model is a first-order transfer function with time constant τ_f . Hence, the fuel processor is simply represented by this first-order model.

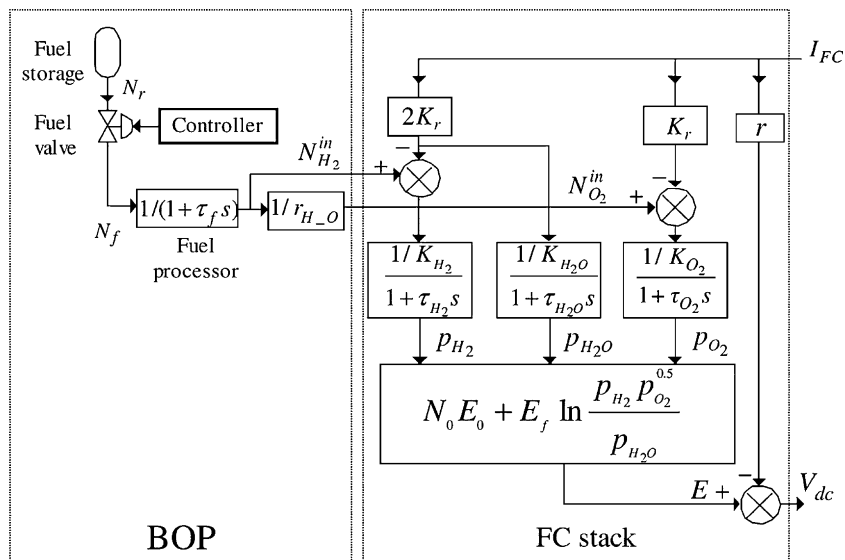


Fig. 1. SOFC dynamic model.

Although CO can be a fuel in the SOFC, we suppose all CO will take part in the CO-shift reaction if the gas contains water [5]. Thus, the overall cell reaction of the SOFC is:



From Eq. (1), it is seen that the stoichiometric ratio between hydrogen and oxygen is 2 to 1. Oxygen excess is always taken in to let hydrogen react with oxygen more completely. So, in this paper the flow ratio of hydrogen to oxygen is kept at 1.145 [16].

2.2. Solid oxide fuel cell

The SOFC consists of hundreds of cells connected in series or in parallel. By regulating the fuel valve, the amount of fuel into the SOFC can be adjusted, and the output voltage of the SOFC can be controlled.

The Nernst’s equation and Ohm’s law determine the average voltage magnitude of the fuel cell stack. Hence, applying Nernst’s equation and Ohm’s law (taking into account ohmic losses), the output voltage of the SOFC can be modeled as follows [16,17]:

$$V_{dc} = E - rI_{FC} \tag{2}$$

$$E = N_0E_0 + \frac{N_0RT}{2F} \ln \frac{p_{H_2}p_{O_2}^{0.5}}{p_{H_2O}} \tag{3}$$

where

$$p_{H_2}(s) = \frac{1/K_{H_2}}{1 + \tau_{H_2}s} (N_{H_2}^{in} - 2K_r I_{FC}) \tag{4}$$

$$p_{O_2}(s) = \frac{1/K_{O_2}}{1 + \tau_{O_2}s} (N_{O_2}^{in} - K_r I_{FC}) \tag{5}$$

$$p_{H_2O}(s) = \frac{1/K_{H_2O}}{1 + \tau_{H_2O}s} 2K_r I_{FC} \tag{6}$$

2.3. Fuel utilization

Fuel utilization is one of the most important operating variables affecting the performance of FC. It is defined as [5]:

$$u = \frac{N_{H_2}^{in} - N_{H_2}^{out}}{N_{H_2}^{in}} = \frac{N_{H_2}^r}{N_{H_2}^{in}} = \frac{N_0 I_{FC}}{2FN_{H_2}^{in}} \tag{7}$$

where $N_{H_2}^r$ is the hydrogen reacted flow rate. When the stack is operated at a high fuel utilization, both the voltage and power density decrease. Furthermore, if fuel utilization is too large, it becomes impossible for the SOFC to sustain the voltage across the load. However, it is a great waste under a low fuel utilization when there is no cycling of the anode gas flow. Therefore, the fuel utilization should be carefully selected to achieve the high SOFC performance. In this analysis, the fuel utilization is varied from 0.7 to 0.9, according to [16,17].

To protect the SOFC stack and avoid large deviations in the terminal voltage due to changes in the stack current, we will hold the utilization constant. According to Eq. (7), the operation of the SOFC stack with a fuel input proportional to the stack current results in a constant utilization factor in the steady state. Thus, the SOFC stack is operated with constant steady-state utilization by controlling the natural gas input flow to the stack as [17]:

$$N_f = \frac{N_0 I_{FC}}{2Fu_s} \tag{8}$$

where u_s is the desired utilization in steady state. Furthermore, because the fuel processor is specially considered in the dynamic modeling of the SOFC, the relationship between a small change of

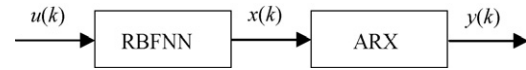


Fig. 2. Hammerstein model.

stack current ΔI_{FC} and a small change of hydrogen input $\Delta N_{H_2}^{in}$ fed to the SOFC stack can be derived as [17]:

$$\Delta N_{H_2}^{in} = \frac{N_0}{2Fu_s(1 + \tau_f s)} \Delta I_{FC} \tag{9}$$

3. Hammerstein model

3.1. Problem statement

In this section, we will consider the problem of estimating a model for a Hammerstein system based on the input–output data, i.e., $\{u_i\}_{i=1, \dots, n}$ and $\{y_i\}_{i=1, \dots, n}$. The Hammerstein model consists of a RBFNN for identification of the static nonlinearity, in series with an ARX model for identification of the linear part. The structure of the Hammerstein model adopted in this paper is illustrated in Fig. 2, where $u(k)$ and $y(k)$ are the input and the output of the Hammerstein model at the k th sampling instant, respectively, and $x(k)$ is the output of the RBFNN which is usually unmeasurable. The output of the Hammerstein model is expressed as:

$$y(k) = -\sum_{i=1}^{n_a} a_i y(k-i) + \sum_{j=0}^{n_b} b_j x(k-j) \tag{10}$$

where a_i ($i=1, \dots, n_a$) and b_j ($j=0, \dots, n_b$) are the parameters of the ARX model, n_a and n_b are integers related to the model order and the function.

The static nonlinear part in the Hammerstein model is represented by using the RBFNN depicted in Fig. 3 as:

$$x(k) = \sum_{i=1}^M w_i \phi_i(u(k)) \tag{11}$$

where

$$\phi_i(u(k)) = \exp\left(-\frac{\|u(k) - c_i\|^2}{2d_i^2}\right) \tag{12}$$

is the Gaussian function. M is the number of hidden node, c_i and d_i are the centers and widths of the i th RBF hidden unit, respectively.

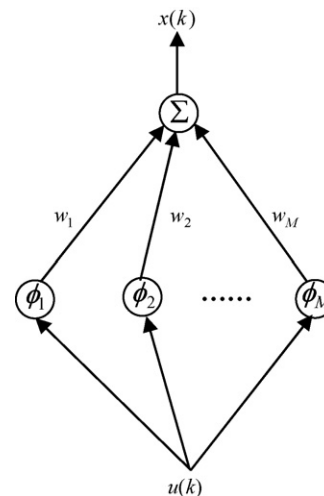


Fig. 3. RBF neural network.

w_i is the connection weights from the i th hidden node to the output, $\|\cdot\|$ denotes the Euclidean norm.

3.2. Identification of the Hammerstein model

Identification of the Hammerstein model involves estimating the hidden centers, the radial basis function widths and the connection weights of the RBFNN, and the orders and the parameters of the ARX model.

The learning procedure of the RBFNN mainly includes two parts, i.e., the hidden centers and the radial basis function widths are determined firstly, followed by the adjustment of the connection weights. The standard RBFNN training method is time-consuming, since it requires examining many different network structures using a trial and error procedure. In addition, the standard training method determines the hidden centers by a clustering approach which usually results in a large number of selected centers [18]. Furthermore, it is frequently observed that error correction learning algorithms, which aim to provide the most appropriate weights, approach to local optima due to their gradient-based attitude.

On the other hand, in most cases of the ARX identification, only the parameters of the ARX model need to be identified and the structure is given beforehand. However, like the other modeling requirements, the model structure is a vital and difficult issue to address [19]. In fact, it is hard to guess a correct model structure for the ARX model if little information of the system is known. An inappropriate model structure will degrade the performance of the ARX identification process badly no matter which algorithm is used to estimate the parameters. Therefore, selecting a proper model structure for the ARX model is as critical as estimating the model parameters.

Genetic algorithm (GA) is a kind of global search algorithm based on the mechanics of natural selection and population genetics. It has been successfully applied to a variety of optimization problems. For instance, it has been employed to identify the parameters of the ARX and non-linear ARX (NARX) models, and to select the optimal model structures of RBFNN and recurrent neural network (RNN). In this study, GA is adopted to optimize the hidden centers, the radial basis function widths and the weights of the RBFNN, and the structure of the ARX model which is entirely defined by the integers n_a and n_b at the same time. After the ARX model structure is determined, the parameters of the ARX model can be estimated by the least squares (LS) algorithm.

3.2.1. Optimization of the RBFNN and the ARX model structure based GA

GA is an iterative stochastic methodology which derives its behavior from the process of evolution in nature. It starts with a random population of possible solutions (chromosomes). The fitness of each chromosome is measured by computing the corresponding value of a fitness function. Then new generations are produced by giving more probabilities of surviving to the individuals with the best fitness values. As the algorithm proceeds, the members of the population are gradually improved. The parallel searching mechanism is the main advantage of the GA, since it is unlikely to get trapped in local minima.

Before optimizing, some GA parameters such as population size P , crossover probability p_c , mutation probability p_m , the maximum numbers of iterations, etc., need to set. The optimizing process of the GA is given as follows:

(1) *Encoding*: Encoding is the first and an important part of the GA process. Binary coding is normally used and has been proved to be optimal in certain cases [20]. In this study, the binary coding scheme is adopted. In encoding, every chromosome includes

the hidden centers, the radial basis function widths and the weights of the RBFNN, and the orders of the ARX model. To obtain the desired accuracy, eight bits binary code is selected to encode each parameter above.

(2) *Fitness function*: The fitness function has a great affect on the convergence speed of the GA process. To derive the GA search process towards the location of the best solution, the fitness function should be able to reflect the key properties of the Hammerstein model. If the fitness function contains inadequate information about the Hammerstein model, it will not be capable of identifying a chromosome with superior characteristics [21]. The fitness function in this paper is chosen as [22]:

$$f_i = J_{\text{worst}} - J_i (i = 1, 2, \dots, P) \quad (13)$$

where J_{worst} implies the Akaike Information Criterion (AIC) for the worst individual, i.e.,

$$J_{\text{worst}} = \max_i J_i \quad (14)$$

Here, J_i denotes the AIC of the i th individual defined by:

$$J_i = N \log(E_i) + 2(n_a + n_b) \quad (15)$$

where

$$E_i = \frac{1}{N} \sum_{k=1}^N (y(k) - \hat{y}^{(i)}(k))^2 \quad (16)$$

where, $\hat{y}^{(i)}(k)$ is the estimated output of the i th individual at time step “ k ” of the Hammerstein model, N is the number of sample points, E_i is the mean square error between the actual output and estimated output of the i th individual.

(3) *Genetic operations*:

Selection: After the fitnesses of all the chromosomes are evaluated, the GA enters the selection phase. According to the fitness values, better solution candidates (individuals) are selected. The most familiar selection procedure is fitness-proportional selection (roulette wheel selection). So the roulette wheel selection strategy is adopted in this study.

Crossover: Crossover is a probabilistic process that exchanges information between two parent chromosomes for generating two offspring. Here single-point crossover with a fixed crossover probability of p_c is selected.

Mutation: Generally, over a period of several generations, the gene pool tends to become more and more homogeneous as one gene begins to dominate. So, a mutation operator is introduced to guard against premature convergence. Mutation randomly alters the gene from 0 to 1 or from 1 to 0 with probability p_m . The mutation probability is set a very small value, usually chosen in the range of 0.001–0.01.

3.2.2. Parameter identification of the ARX model

After the ARX model structure is determined, the LS algorithm is adopted to estimate the parameters of the ARX model, i.e.,

$$\hat{\theta} = [H^T H]^{-1} H^T Y \quad (17)$$

where $\hat{\theta} = [\hat{a}_1, \hat{a}_2, \dots, \hat{a}_{n_a}, \hat{b}_0, \hat{b}_1, \dots, \hat{b}_{n_b}]^T$ is the estimated parameters of the ARX model and

$$H = [h(1), h(2), \dots, h(N)]_{N \times (n_a + n_b + 1)}^T \quad (18)$$

$$Y = [y(1), y(2), \dots, y(N)]_{N \times 1}^T \quad (19)$$

$$h(k) = [-y(k-1), -y(k-2), \dots, -y(k-n_a), x(k), x(k-1), \dots, x(k-n_b)]_{(n_a+n_b+1) \times 1}^T \quad (20)$$

4. Model predictive control

MPC refers to a class of control algorithms in which a dynamic model of the plant is used to predict and optimize the future behavior of the process. The basic control strategy of the MPC is to select a set of future control horizons and minimize a cost function based on the desired output trajectory over a prediction horizon with a chosen length.

4.1. Output voltage prediction

The MPC predicts the future output voltage of the SOFC with the Hammerstein model and past input–output data in this study.

Based on the established Hammerstein model, the predictive output voltage $\hat{V}_{dc}(k+l)$ for p steps ahead can be obtained by successive iterations:

$$\hat{V}_{dc}(k+l) = -\sum_{i=1}^{n_a} a_i \hat{V}_{dc}(k+l-i) + \sum_{j=0}^{n_b} b_j x(k+l-j), \quad 0 \leq l \leq p \quad (21)$$

where

$$x(k) = \sum_{i=1}^M w_i \exp\left(-\frac{\|N_f(k) - c_i\|^2}{2d_i^2}\right) \quad (22)$$

4.2. Reference trajectory

The control goal of the SOFC is that its output voltage can reach the set point quickly and smoothly. The output voltage of the SOFC should follow a reference trajectory:

$$V_{dcr}(k+j) = \alpha^j V_{dc}(k) + (1 - \alpha^j) V_{sp} \quad (23)$$

where $V_{dcr}(k+j)$ is the reference trajectory of the output voltage at time $k+j$, $V_{dc}(k)$ is the actual output voltage at time k , V_{sp} is the set point of the output voltage, $j=1, 2, \dots, p$ are coincidence points, and α is a parameter ($0 < \alpha < 1$).

4.3. Optimization criteria

The objective of the MPC controller is to compute future control moves which will minimize a criterion function based on the desired output trajectory over a prediction horizon H_p . In predictive controllers, the predictive control law can be yielded by minimizing the cost function. Hence, the choice of the criterion function is of paramount importance [23]. To obtain the appropriate control law, the following criterion function is minimized. The criterion function is a quadratic function of the errors between the predicted output voltage and the desired trajectory (reference) over the prediction horizon, and includes terms which penalize the excessive changes in the manipulated variables (the natural gas input flow).

$$J = \sum_{j=1}^{H_p} [\hat{V}_{dc}(k+j) - V_{dcr}(k+j)]^2 + \lambda \sum_{i=1}^{H_c} [N_f(k+i) - N_f(k+i-1)]^2 \quad (24)$$

subject to:

$$N_{f, \min} \leq N_f \leq N_{f, \max}$$

where $\hat{V}_{dc}(k+j)$ is the predicted output voltage of the SOFC, H_c is the control horizon, $N_f(k+i)$ is the manipulated variable at time $k+i$, and λ is the weighting factor.

The control signal (the natural gas input flow) changes only inside the control horizon and remains constant afterward, i.e.,

$$N_f(k+j) = N_f(k+H_c-1), \quad j = H_c, \dots, H_p \quad (25)$$

5. Results

5.1. Hammerstein model of the SOFC

To establish the desired Hammerstein model, we choose the natural gas input flow, the oxygen flow, the operating temperature, and the stack current as the model inputs, the voltage as the output. For keeping the utilization constant, the natural gas input flow can be controlled according to the stack current which is proportional to the terminal load. A decrease in the load current will in turn increase the output voltage of the SOFC, while the decrease in the current decreases the natural gas input flow.

For the purpose of identification, the white-box model described in Section 2 is excited with band-limited white noise around the nominal value of the natural gas input flow (N_f), while all the other inputs are kept constant in their nominal values. The nominal operating conditions of the SOFC are given in Table 1 [5,16]. Changes in the natural gas input flow are produced every 10 s, with a maximum amplitude of $\pm 70\%$ of the nominal value. As a result, a set of 6000 data points is collected from the simulation with a sampling time of 0.1 s. The first 3000 data are used to estimate the Hammerstein model of the SOFC, while the following 3000 data are used for validation purposes.

Through a large number of tests, a RBFNN with 6 hidden nodes in the Hammerstein model is chosen for its better performance. The GA parameters are chosen as: population size $P=40$, the maximum number of iterations = 50, the total length of every chromosome is $3 \times 6 \times 8 + 2 \times 8$, $p_c = 0.6$ and $p_m = 0.008$. When a load disturbance causes the stack current to have a step change (from 300 A to 270 A) at 150 s, the actual output voltage and the predicted output voltage of the Hammerstein model are represented in Fig. 4 by using the above identification algorithm. Fig. 4 indicates that the proposed Hammerstein modeling method is applicable to describe the nonlinear dynamic behaviors of the SOFC, and the Hammerstein model of the SOFC presented in this paper is accurate and valid.

5.2. Fuel utilization and output voltage control

According to Eq. (8), we can regulate the natural gas input flow to hold the utilization as steady-state constant in the presence of external disturbances. To prove the validities of the control strategies, we choose the current disturbance as a multiple step

Table 1
SOFC operating point data

Item	Value
N_0	384
T	1273 K
$I_{FC,rate}$	300 A
u_s	0.8
E_0	1.18 V
K_{H_2}	$0.843 \text{ mol s}^{-1} \text{ atm}^{-1}$
K_{O_2}	$2.52 \text{ mol s}^{-1} \text{ atm}^{-1}$
K_{H_2O}	$0.281 \text{ mol s}^{-1} \text{ atm}^{-1}$
τ_{H_2}	26.1 s
τ_{O_2}	2.91 s
τ_{H_2O}	78.3 s
τ_f	5 s
r	0.126 Ω
r_{H_2O}	1.145

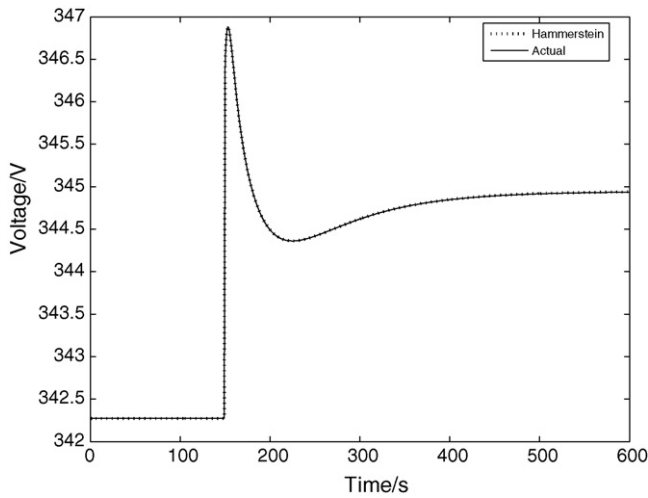


Fig. 4. Output voltage of the actual and identified Hammerstein models.

signal which reduces from 300 A to 270 A at 150 s, and goes back to 300 A after 650 s. In this situation, the control effect of the fuel utilization is depicted in Fig. 5. From Fig. 5, we can see the fuel utilization of the SOFC can be controlled as steady-state constant by regulating the natural gas input flow according to the stack current.

Based on the Hammerstein model, the MPC scheme can be developed in succession. The objective of the MPC study is to maintain the output voltage of the SOFC as a desired value after the fuel utilization is kept constant. The parameters for the MPC strategy are chosen as: $H_p = 40$, $H_c = 6$ and $\alpha = 0.3$. The proposed Hammerstein model-based MPC algorithm is simulated and the result is shown in Fig. 6, which clearly demonstrates the excellent performance of the MPC scheme.

For the purposes of comparison, the voltage control result of the SOFC using PI controller proposed in [16] is depicted in Fig. 7. Comparing Fig. 6 with Fig. 7, one will notice that the output voltage of the SOFC can achieve the desired value no matter which controller above is adopted. In the case of the above current disturbances, the output voltage using the MPC controller can achieve the desired value in repose. However, it is more fluctuant when the PI controller is used to control the output voltage.

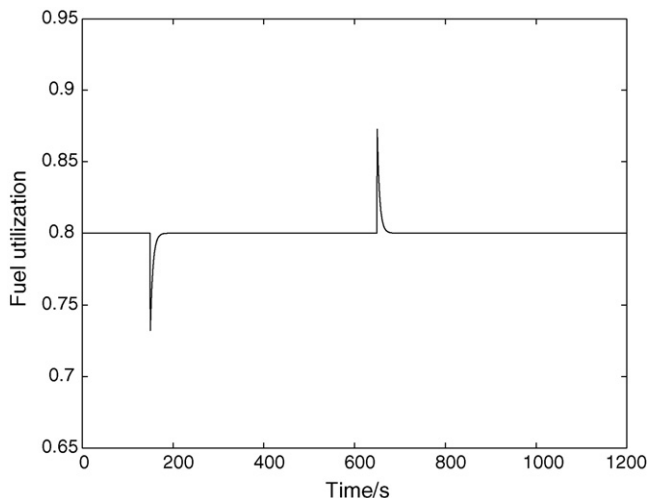


Fig. 5. Fuel utilization control of the SOFC.

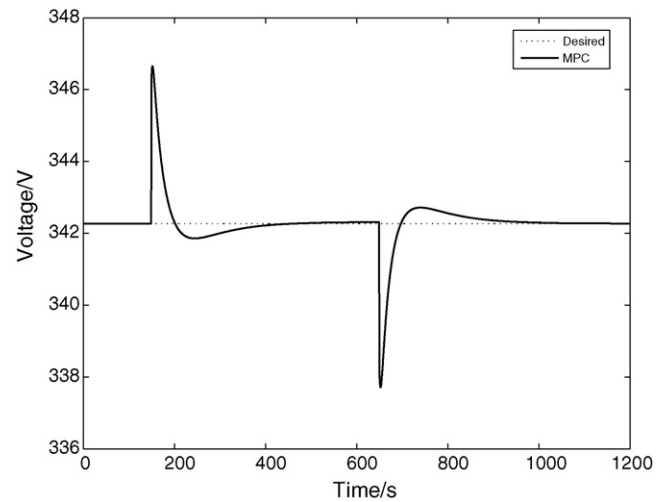


Fig. 6. Voltage control of the SOFC using MPC controller.

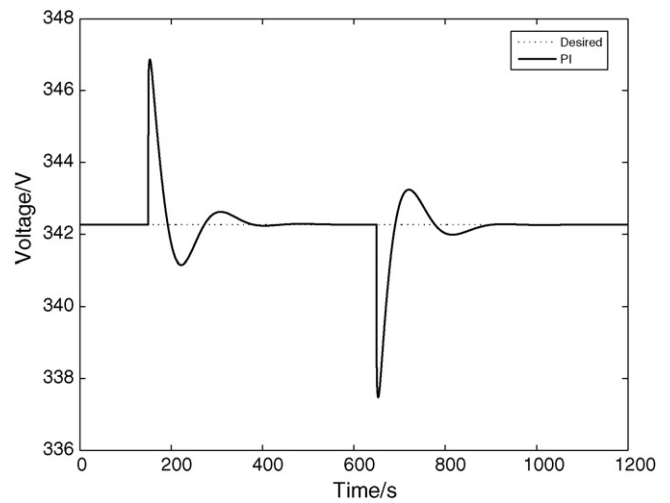


Fig. 7. Voltage control of the SOFC using PI controller.

To evaluate the performance of the control systems, the integral of time absolute error (ITAE) is used:

$$ITAE = \sum_{k=1}^{1200} k|V_{dc}(k) - V_{sp}(k)| \quad (26)$$

The resulting ITAE using MPC is $1.4015e+005$, which is smaller than that using PI controller ($1.5762e+005$). This indicates that the MPC scheme based on the Hammerstein model has better control effects than that of the PI controller developed in [16].

6. Conclusions

To develop valid control strategies, a Hammerstein model of the SOFC has been developed to describe the nonlinear dynamic behaviors of the output voltage and the natural gas input flow accurately. The Hammerstein model adopted in this paper consists of the RBFNN in series with the ARX model. Simulation results have illustrated the applicability of the proposed Hammerstein model in modeling the nonlinear dynamic properties of the SOFC.

For protecting the SOFC, a primary control loop has been designed to hold the fuel utilization as steady-state constant. Then based on the Hammerstein model, a nonlinear MPC controller has

been developed to control the output voltage of the SOFC at a desired value by regulating the natural gas input flow. The good control effect of the fuel utilization and the favorable performance of the MPC controller for voltage control of the SOFC have been testified in this study.

It is noteworthy that in the Hammerstein modeling the operating temperature of the SOFC has been assumed as constant and the details associated with concentration variations have not been reported. In future works, a control oriented SOFC model considering temperature effects and the details associated with concentration variations will be developed. Furthermore, based on this model, some control strategy studies can be pursued.

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